JOURNAL OF ECONOMIC THEORY **71**, 349–377 (1996)
ARTICLE NO. 0125

The Meaning and Measurement of Income Mobility*

Gary S. Fields

School of Industrial and Labor Relations, Cornell University, Ithaca, New York 14853

and

Efe A. Ok

Department of Economics, New York University, 269 Mercer Street, New York, New York 10003

Received September 2, 1994; revised March 1, 1996

Income mobility may be seen as arising from two sources: (i) the transfer of income among individuals with total income held constant, and (ii) a change in the total amount of income available. In this paper, we propose several sensible properties defining the concept of income mobility and show that an easily applicable measure of mobility is uniquely implied by these properties. We also show that the resulting measure is additively decomposable into the two sources listed above, namely, mobility due to the transfer of income within a given structure and mobility due to economic growth or contraction. Finally, these results are compared and contrasted with other mobility concepts and measures in the literature. *Journal of Economic Literature*: Classification Numbers: D31, D63. © 1996 Academic Press, Inc.

1. INTRODUCTION

One of the important issues that can be studied with longitudinal data, and only with longitudinal data, is that of income mobility.[1] Indeed, as more longitudinal data sets become available for an increasing number of

* An earlier version of this paper was presented at the Northeastern Universities Development Economics Conference, the 2nd International Meeting of the Society for Social Choice and Welfare, the Séminaire René Roy, the Paris Seminar on Economic Justice, the Université Catholique de Louvain, the Universitat Autónoma de Barcelona, Boston College, Cornell and Yale Universities. In addition to seminar participants, the authors wish to thank Richard Arnott, Roland Benabou, François Bourguignon, Joan-Maria Esteban, Roger Guesnerie, Robert Hutchens, Peter Lambert, Tapan Mitra, Oscar Rothaus, Anthony Shorrocks, Alain Trannoy, and William Thomson for constructive criticism and insightful suggestions. We also thank the two anonymous referees of this journal, whose comments improved the paper substantially. The support of the Center for Analytic Economics at Cornell University is also gratefully acknowledged.

[1] Although our discussion will be in terms of "incomes" among "individuals", any real-valued measure of socioeconomic position (e.g., consumption, earnings, occupational status index) among any well-defined recipient unit (e.g., households, workers, generations, per capitas, adult equivalents) would do.

0022-0531/96 $18.00

countries, the literature on the measurement of income mobility has become quite extensive in the last two decades.

The purpose of this paper is to derive a theoretically justified measure of absolute income mobility which can be applied to longitudinal data. We ask and attempt to answer the following three questions: (1) Has income mobility (movement) taken place in a given economy over time, and if so, how much? (2) When does one economy, group, or time period exhibit more income variation than another? (3) What are the sources of income mobility?

To address these questions properly, it is necessary to clarify what is meant by *income mobility*, a concept on which there is hardly agreement either conceptually or practically. The approach followed in this paper is to specify some properties which an index of (absolute) income mobility that focuses on the variation of individual incomes should satisfy, and then to use these as axioms to characterize a mobility measure. Since it will be shown that this characterization analysis yields a unique measure, one can view what the resulting index measures as equivalent to the definition of *absolute income mobility*. In consequence, there can be no debate on whether we are successfully measuring what we want to or not, and conversely, if one dislikes the derived measure, then we immediately know that the fundamental dispute is about the basic conception of absolute income movement/mobility, not about the way we intend to measure it.

Another aim of the present work is to demonstrate how the sources of (absolute) mobility can be gauged by disaggregating this new measure. In doing this, we shall regard income mobility as arising from two sources: transfer of money among individuals with total income held constant, and changes in the total income available.

To illustrate the first of these, suppose we have three individuals with incomes \$1, \$2, and \$3, so the initial income distribution is (1, 2, 3). Let the third individual transfer \$2 to the first individual so that the resulting distributional change can be denoted as $x = (1, 2, 3) \to y = (3, 2, 1)$. Unlike the standard literature on income inequality, in which *anonymity* is a fundamental assumption, in the present context, it matters which individuals have what amount of income. Income mobility has taken place in $x \to y$ precisely because money has changed hands. More generally, holding the total income constant, the larger is one person's income gain (and the larger is another's loss), the more mobility (movement) there is. So, for instance, there would be more mobility in $(1, 2, 3) \to (4, 2, 0)$ than in $(1, 2, 3) \to (3, 2, 1)$.

The second source of income mobility arises because of changes in the total amount of income available. As a trivial example, one can think of a Robinson Crusoe economy where Crusoe's income shifts from \$0 to \$100 in a given period. We would say that some mobility has taken place (although the relative position of Crusoe is vacuously the same). To give another example, let us start with an initial distribution of income given by

$x=(1, 1, 1, 1, 1, 1, 2)$, and suppose that one additional high-income position is created and filled by one of the previously low-income individuals: $x=(1, 1, 1, 1, 1, 1, 2) \rightarrow y=(1, 1, 1, 1, 1, 2, 2)$. (Absolute) income mobility has taken place because of the creation of an additional high-income position. If even more high-income positions had been created, say $(1, 1, 1, 1, 1, 1, 2) \rightarrow z=(1, 1, 1, 1, 2, 2, 2)$, there would have been more income mobility. Development economists call this process *modern sector enlargement*, and agree that there is more of it in going from x to z than from x to y.

In passing, we note that mobility and inequality comparisons can move in completely different directions. For instance, while $(1, 2, 3)$ and $(3, 2, 1)$ are clearly equally unequal, it is difficult to dispute that the process $(1, 2, 3) \rightarrow (3, 2, 1)$ exhibits a positive amount of income variation. To give another example, notice that there is one dollar worth of income growth in both A: $(1, 2, 3) \rightarrow (1, 2, 4)$ and in B: $(1, 2, 3) \rightarrow (2, 2, 3)$, and a reference to impartial treatment of individuals might urge one to conclude that the (absolute) mobility (movement) depicted in the processes A and B are the same, yet A represents an unambiguous increase in inequality whereas B has an unambiguous decrease in inequality. The distinction between income inequality and what we mean here by "mobility" should be kept in mind in what follows.

This paper seeks to derive a measure of (absolute) income mobility which focuses on *aggregate income movement*, and which encompasses both mobility due to the transfer of income and mobility due to growth and to justify the choice of a mobility measure on fundamental grounds. The organization of the rest of the paper is as follows. In Section 2, we discuss a number of properties which are guided by the two sources of mobility described above, and which, we believe, characterize income mobility. Taking these properties as axioms, we derive in Section 3 a new, very easily applicable mobility measure which is uniquely consistent with the specified set of axioms. In this sense, what we *intend to* measure and what we *actually* measure coincide. In Section 4, we demonstrate that our measure of total mobility is additively decomposable into two parts, one attributable to *transfer mobility* and the other attributable to *growth mobility*. This shows rigorously that the above mentioned sources are the only determinants of our conception of mobility. Section 5 is devoted to a discussion of the relationship between our approach and others in the literature. We close with a concluding section. An appendix supplies the proofs of the main results stated in the text.

2. AXIOMS FOR A MEASURE OF INCOME MOBILITY

Take $\mathbf{R}_{+}^{n}$ as the space of all income distributions with population $n \geqslant 1$. Let $x=(x_1, x_2, ..., x_n) \in \mathbf{R}_{+}^{n}$, where x_j corresponds to the income level of the jth person, $j=1, 2, ..., n$. Suppose x becomes $y \in \mathbf{R}^n_+$, where the individuals are ordered the same in y as in x: $x \rightarrow y$. Asking how much mobility has taken place might be rephrased as how much apart x and y have become for an appropriate distance function $d_n: \mathbf{R}^n_+ \times \mathbf{R}^n_+ \rightarrow \mathbf{R}_+$. In this interpretation, $d_n(x, y)$ stands for the *total* (*absolute*) *income mobility* that is observed in $x \rightarrow y$. (See Cowell [14] for a similar approach.) In what follows, we shall make assumptions concerning the nature of $\{d_n: \mathbf{R}^{2n}_+ \rightarrow \mathbf{R}_+ \mid n \geqslant 1\}$.[2] (For expositional ease, this class is referred to as d_n throughout the text.)

Before stating the axioms, it should be stressed that we view d_n as a measure of *total* mobility in a population of n people and the following axioms should be considered with this in mind. However, this might cause problems when the sizes of the groups being analyzed vary, e.g. as between one longitudinal survey and another, or between one group and another within the same longitudinal survey. Therefore, in empirical applications, one might also be interested in *per capita* and/or *percentage* mobility measurement. Given $x \rightarrow y$, $x, y \in \mathbf{R}^n_+$, $n \geqslant 1$, accepting d_n as a total mobility index, the per capita measure would be defined as

$$m_n(x, y) \equiv \frac{d_n(x, y)}{n}, \tag{1}$$

and the percentage measure would take the form

$$p_n(x, y) \equiv \frac{d_n(x, y)}{\sum_{j=1}^{n} x_j}. \tag{2}$$

Taking these as definitions, axiomatizing d_n amounts to axiomatizing m_n and p_n as well. In other words, the characterization of the total mobility index d_n will implicitly yield a characterization for both the per capita mobility index m_n and the percentage mobility index p_n.

We now proceed to a discussion of the set of axioms that will characterize d_n.

Axiom 2.1 (*Linear homogeneity*). For $n \geqslant 1$, $d_n(\lambda x, \lambda y) = \lambda d_n(x, y)$ for all $\lambda > 0$, $x, y \in \mathbf{R}^n_+$.

Axiom 2.2 (*Translation invariance*). For $n \geqslant 1$, $d_n(x + \alpha \mathbf{1}_n, y + \alpha \mathbf{1}_n) = d_n(x, y)$ for all $\alpha > 0$, $x, y \in \mathbf{R}^n_+$ such that $x + \alpha \mathbf{1}_n, y + \alpha \mathbf{1}_n \in \mathbf{R}^n_+$, where $\mathbf{1}_n \equiv (1, 1, ..., 1) \in \mathbf{R}^n$.

Axiom 2.3 (*Normalization*). $d_1(1, 0) = d_1(0, 1) = 1$.

[2] Although we motivate the function d_n as a distance function, the usual distance properties are not, in fact, postulated on it. Therefore, our only basic premise is that the mobility observed in $x \rightarrow y$, $x, y \in \mathbf{R}^n_+$, is a function of x and y.

The first two axioms are the standard assumptions of the theory of economic distances (see, e.g., Ebert [16], Chakravarty and Dutta [11] and Chakravarty [10].) Axiom 2.1 indicates that d_n is scale dependent, that is, an equiproportional change in all income levels (both in the initial and final distributions) results in exactly the same percentage change in the mobility measure. Axiom 2.2, being a straightforward reflection of Kolm's leftist inequality criterion (cf. [25]), is a base independence requirement. It states that, given the amount of mobility found in going from one distribution to another, if the same amount is added to everybody's income in both the original and the final distributions, the new situation has the same mobility as the original one. Axiom 2.3 says that a one dollar income gain and a one dollar income loss both produce one unit of mobility for that individual.

Remarks. (i) These three axioms characterize $d_1 : \mathbf{R}^2_+ \to \mathbf{R}_+$ uniquely as $d_1(x, y) = |x - y|$ for all $x, y \geqslant 0$.

(ii) Axiom 2.2 clarifies that d_n cannot be viewed as a measure of *percentage* mobility.

(iii) Axiom 2.2 reveals that we are interested in *absolute* mobility as opposed to *relative* mobility. For instance, when examining relative mobility measures, Shorrocks [42] postulates a scale invariance property whereby $d_n(x, y) = d_n(\lambda x, \lambda y)$ for all $\lambda > 0$, $x, y \in \mathbf{R}^n_+$. This property clearly contradicts Axiom 2.1 and thus none of the measures with this property (e.g., the Hart, Shorrocks, and Maasoumi–Zandvakili indices) can be used as d_n (see [42, Table 1, p. 20]). For example, consider the following two transformations: A: $(1, 2) \to (100, 200)$ and B: $(2, 4) \to (200, 400)$. Any relative income mobility measure, by definition, would record the same amount of mobility in A as in B. On the other hand, an absolute income mobility measure which satisfies Axiom 2.1 would obviously see twice as much mobility in B as in A.

Axiom 2.4 (*Strong decomposability*). For $n \geqslant 2$ and all $x^i, y^i \in \mathbf{R}^{n_i}_+$, $i = 1, 2$, with $n_1 + n_2 = n$,

$$d_n(x, y) = F_n(d_{n_1}(x^1, y^1), d_{n_2}(x^2, y^2)),$$

for some symmetric, nonzero and continuous $F_n : \mathbf{R}^2_+ \to \mathbf{R}_+$.[3]

This axiom is in a similar vein with the standard formulations of the "decomposability" property in the context of income inequality measurement (cf. Bourguignon [8] and Shorrocks [41], *inter alia*). It simply posits

[3] We gratefully owe this version of Axiom 2.4 to an anonymous referee of this journal.

that the level of total income mobility a population experiences is a nontrivial function of the levels of mobility experienced by any two disjoint and exhaustive subpopulations. While the continuity of this function is a natural regularity condition, the symmetry property of it assures the impartial treatment of each subgroup.

However, we should note that strong decomposability is a rather demanding property. Given that transformations $x^1 \to y^1$ and $x^2 \to y^2$ are combined to $(x^1, x^2) \to (y^1, y^2)$, it requires that the aggregate mobility be a function of *only* the levels of mobility observed in the subtransformations, and not, for instance, of the sizes of the subgroups. This is actually a stronger requirement than the approach followed in the theory of inequality decomposition. It would be worthwhile, therefore, to consider the following weakening of Axiom 2.4:

Axiom 2.4* (*Weak decomposability*). For $n \geqslant 2$ and all $x, y \in \mathbf{R}^n_+$,

$$d_n(x, y) = G_n(d_1(x_1, y_1), d_1(x_2, y_2), ..., d_1(x_n, y_n)),$$

for some symmetric, nonzero and continuous $G_n: \mathbf{R}^n_+ \to \mathbf{R}_+$.[4]

Weak decomposability asserts that total income mobility is a nontrivial function of the observed changes in the income levels of the constituent individuals. It certainly carries the same spirit as the symmetry, decomposability and smoothness assumptions of Cowell [14] while being technically weaker. Since the intuitive support for the assumptions of continuity and symmetry is straightforward, it appears natural to view Axiom 2.4* as a rather innocuous postulate.

Our next axiom is an independence condition:

Axiom 2.5 (*Population consistency*). Let $n, m \geqslant 2$, $x, y \in \mathbf{R}^n_+$ and $z, w, \in \mathbf{R}^m_+$. Then,

$$d_n(x, y) = d_m(z, w) \quad \text{implies} \quad d_{n+1}((x, \alpha), (y, \beta)) = d_{m+1}((z, \alpha), (w, \beta)),$$

for any $\alpha, \beta \geqslant 0$.

In effect, what this axiom says is that in the context of populations of different sizes, if equals are added to equals the results are equal. Let $x \to y$ be observed in a population of n individuals and let $z \to w$ be observed in another population of m individuals. Suppose that the level of income mobility is judged to be the same in the two situations. Axiom 2.5 says that if an agent with initial income level $\alpha \geqslant 0$ and final income level $\beta \geqslant 0$ is added to both situations, then the two should still have the same mobility.

[4] One can easily see that Axiom 2.4 implies Axiom 2.4*. On the other hand, there are indices which satisfy Axioms 2.1, 2.2, 2.3 and 2.4* but not Axiom 2.4. An example is: $d_n(x, y) = \frac{1}{n}(\sum_{j=1}^n |x_j - y_j|)$.

In other words, if $x \to y$ has the same mobility as $z \to w$, then $(x, \alpha) \to (y, \beta)$ has the same mobility as $(z, \alpha) \to (w, \beta)$ for any $\alpha, \beta \geqslant 0$.

We view Axiom 2.5 as a reasonable assumption for a *total* mobility index to satisfy. Since it acts rather as an independence assumption with respect to the size of the population, one may initially find this postulate unattractive, however. For example, a mobility measure of the form $d_n(x, y) \equiv K(\sum_{j=1}^n |x_j - y_j|, n)$ satisfies Axiom 2.5 only if $K(\cdot, n) = K(\cdot, m)$ for all $n, m \geqslant 2$. But such an objection does not carry much weight. We would indeed like to eliminate a measure like $d_n(x, y) = K(\sum_{j=1}^n |x_j - y_j|, n)$ unless K is not constant in the second argument, for such an index (with K being non-constant in the second argument) cannot be justly viewed as a *total* mobility index. Indeed, it seems reasonable to think of an index d_n as a *total* mobility index if it guarantees that adding an individual to the population never decreases the level of mobility, and leaves mobility unaltered if the individual income change of the new agent is nil. For instance, there is a clear sense in which a measure like $d_n(x, y) \equiv 1/n \sum_{j=1}^n |x_j - y_j|$ (or more generally, a measure like $d_n(x, y) \equiv K(\sum_{j=1}^n |x_j - y_j|, n)$ with K non-constant in the second argument) is not a satisfactory measure of *total* mobility, for, we would argue, it should rather be viewed as a (good or bad) *per capita* measure of income mobility. (The analogy with the familiar notions of *total* GNP and *per capita* GNP is straightforward.) Looked at this light, Axiom 2.5 might not seem like an unacceptable postulate. (Furthermore, we emphasize that population consistency is a consequence of strong decomposability (Axiom 2.4) provided that Axioms 2.1, 2.2 and 2.6 hold.)

Axiom 2.6 (*Growth sensitivity*). Let $n \geqslant 1$ and $x, y, z, w \in \mathbf{R}_+^n$. If, for any k, $1 \leqslant k \leqslant n$,

$$d_1(x_j, y_j) = d_1(z_j, w_j) \quad \text{for all} \quad j \neq k \qquad \text{and} \qquad d_1(x_k, y_k) \neq d_1(z_k, w_k),$$

then $d_n(x, y) \neq d_n(z, w)$.

The essence of this axiom is that if unequals are added to equals, the results are unequal. Put precisely, it says that if all individuals except one have the same mobility in two situations but that one individual experiences more mobility in one situation than another, then the two situations must have different levels of mobility. This axiom is identical with Axiom 2.5 of Ebert [16] and seems fairly unexceptionable.

We come now to our final axiom:

Axiom 2.7 (*Individualistic contribution*). Let $n \geqslant 2$. For any $x, y, x', y' \in \mathbf{R}_+^n$ such that $x_1 = x'_1$ and $y_1 = y'_1$, we have

$$d_n(x, y) - d_n(x, (x_1, y_2, ..., y_n)) = d_n(x', y') - d_n(x', (x'_1, y'_2, ..., y'_n)).$$

It may seem plausible that regardless of whether a given individual's income change is counted in dollars, squared dollars, logged dollars, or whatever, the absolute contribution of that individual's income change to the total mobility in the economy should be independent of how it is that *other* people's incomes change. What Axiom 2.7 states is that the contribution of one income recipient's income change to total mobility depends *only* on the amount of his/her income change; by implication, it is independent of the other's income changes.[5]

Nevertheless, Axiom 2.7 appears to be considerably more demanding than the previous axioms we have considered for a mobility measure. Indeed, it implies a strong form of separability and excludes great many functional forms. For this reason, we shall start exploring the implications of the axioms put forth above without invoking the postulate of individualistic contribution at first. This yields a class of mobility measures with uncountably many members, and the usefulness of Axiom 2.7 shows itself at this point. Including this postulate into our axiom set results in characterizing a *unique* income mobility measure. So, the result one gets from such an admittedly strong axiom is also a very strong one. Since individualistic contribution is not merely a mathematical requirement, but rather possesses a clear economic interpretation, this may be viewed as a positive virtue.

3. MEASUREMENT OF INCOME MOBILITY

Our first result is a complete characterization of the class of measures induced by Axioms 2.1–6.

THEOREM 3.1. *d_n satisfies Axioms 2.1–2.4 and 2.6 if, and only if, there exists an $\alpha > 0$ such that*

$$d_n(x, y) = \left(\sum_{j=1}^{n} |x_j - y_j|^{\alpha} \right)^{1/\alpha} \qquad \textit{for all} \quad x, y \in \mathbf{R}^n_+ .^6$$

In fact, by replacing Axiom 2.4 with Axioms 2.4* and Axiom 2.5, we obtain precisely the same characterization:

[5] Of course, a given individual's *percentage* contribution to total mobility clearly should be a function of other people's income changes; notice that p_n (recall (2)) allows for this when d_n satisfies Axiom 2.7.

[6] Recall that Axiom 2.4 (and Axiom 2.5) is actually posited on the class $\{d_n : \mathbf{R}^{2n}_+ \to \mathbf{R}_+ \mid n \geqslant 1\}$. Consequently, more precisely put, this theorem yields a characterization of the class $\{d_n : (x, y) \mapsto \sum_{j=1}^{n} |x_j - y_j| \mid n \geqslant 1\}$.

THEOREM 3.2. *d_n satisfies Axioms* 2.1–2.3 *and* 2.4*–2.6 *if, and only if, there exists an* $\alpha > 0$ *such that*

$$d_n(x, y) = \left(\sum_{j=1}^{n} |x_j - y_j|^{\alpha} \right)^{1/\alpha} \qquad \textit{for all} \quad x, y \in \mathbf{R}^n_+.$$

Although these results generate an interesting class of total mobility measures, one has to pick a certain member of this class (that is, choose a certain $\alpha > 0$) in practical computations. Although any such choice will result in a mobility measure which would satisfy Axioms 2.1–2.6, it will also imply a further specification of the notion of income mobility beyond what is captured by these axioms.[7] In other words, an arbitrary specification of α might disguise the basic premise behind the induced mobility measure. However, at least in one particular case, we do not run into this difficulty. Indeed, adding Axiom 2.7 to our set of postulates easily reduces the large class of measures introduced in Theorem 3.1 to a singleton. In fact,

PROPOSITION 3.3. *Let* $d_n(x, y) \equiv f(\sum_{j=1}^{n} f^{-1}(d_1(x_j, y_j)))$ *for all* $x, y \in \mathbf{R}^n_+$, $n \geqslant 2$, *for some continuous and strictly increasing* $f: \mathbf{R}_+ \to \mathbf{R}_+$ *such that* $f(0) = 0$, *and assume that* $d_1(a, a) = 0$ *for all* $a \geqslant 0$. *Then,* d_n *satisfies Axiom* 2.7 *if, and only if,* f *is linear.*

The following is an obvious consequence of Theorems 3.1, 3.2 and Proposition 3.3:

PROPOSITION 3.4. *d_n satisfies Axioms* 2.1–2.4, 2.6 *and* 2.7 (*or, Axioms* 2.1–2.3 *and* 2.4*–2.7) *if, and only if,*

$$d_n(x, y) = d_n^{\circ}(x, y) \equiv \sum_{j=1}^{n} |x_j - y_j| \qquad \textit{for all} \quad x, y \in \mathbf{R}^n_+. \tag{3}$$

Remarks. (i) One might think that to derive an economic distance function axiomatically, the usual metric axioms should be explicitly used (see Shorrocks [40] and Ebert [16]). However, the only axioms that are posited on d_n in the characterization theorem above are Axioms 2.1–2.7; that is, the fact that d_n° satisfies the distance axioms is a result, not an assumption. We view this as one of the good features of our set of axioms, since why an economic distance function should satisfy the triangle inequality as an axiom is not entirely clear.

(ii) Due to its linearity feature, d_n° is additively subgroup decomposable in the sense that the total mobility in the full population is the sum of the total mobilities of each of the subpopulations.

[7] See Mitra and Ok [32] for an alternative approach, however.

(iii) Ebert [16] derives a class of statistical measures of distance between two income distributions, and his main result is rather similar to this theorem. However, the idea developed in that paper does not correspond to a distance function measuring *absolute income mobility* since anonymity is substantially used in Ebert's axiomatization.

(iv) It may be of technical interest that one can replace Axiom 2.7 by a smoothness assumption on the function G_n (recall Axiom 2.4*) and retain the characterization result stated in Proposition 3.4. We have shown elsewhere that if d_n satisfies Axioms 2.1–2.3, 2.5 and 2.6, and if, for all $n \geqslant 2$ and all $x, y \in \mathbf{R}^n_+$,

$$d_n(x, y) = G_n(d_1(x_1, y_1), d_1(x_2, y_2), ..., d_1(x_n, y_n)),$$

for some symmetric, nonzero and continuously differentiable $G_n: \mathbf{R}^n_+ \to \mathbf{R}_+$ such that $\partial G_n / \partial a_j \neq 0$ for all $j = 1, ..., n$, then $d_n = d_n^\circ$ obtains. (See Fields and Ok [18].)

As noted in Section 2, a characterization of the *total* mobility measure d_n° also entails a characterization of both the *per capita* mobility measure m_n° and the *percentage* mobility measure p_n°, via (1) and (2) respectively:

$$m_n^\circ(x, y) \equiv \frac{1}{n} \sum_{j=1}^{n} |x_j - y_j| \qquad \text{for all} \quad x, y \in \mathbf{R}^n_+$$

and

$$p_n^\circ(x, y) \equiv \frac{\sum_{j=1}^{n} |x_j - y_j|}{\sum_{j=1}^{n} x_j} \qquad \text{for all} \quad x, y \in \mathbf{R}^n_+.$$

Notice that, gauging per capita and percentage mobility this way, two societies with the same total mobility but different population sizes would have different per capita mobility, and two societies with the same per capita mobility but different base incomes would exhibit different percentage mobility.[8]

[8] For a fruitful implementation of the measures introduced above, d_n°, m_n° and p_n°, one may have to distinguish between time series and cross country studies. If the empirical comparisons of mobility are to be made across time periods within a country, income changes should be measured in terms of constant units of the country's currency, and if they are to be made across countries, the currencies need to be expressed in the same units by using the most appropriate exchange rate or purchasing power parity conversion.

4. DECOMPOSITION OF TOTAL MOBILITY

The literature on the sources of mobility owes much to the work of Markandya, who noted:

> Within the sociological literature a distinction is made between changes in mobility that can be attributed to the increased availability of positions in higher social classes and those changes that can be attributed to an increased intergenerational movement among social classes, *for a given distribution of positions among these classes.*[9]

Markandya [30] proposed two alternative procedures for decomposing total mobility into components due to structural mobility (the first type of mobility noted in the quotation above) and to exchange mobility (the latter type). Markandya's "Definition I" defines exchange mobility as "that proportion of the change in welfare that could have been obtained without any change in the distribution of incomes. Structural mobility would then be defined as the *balance* of the change in welfare." By contrast, "Definition II" "define[s] the change in welfare that would have taken place with a completely immobile transition matrix as structural mobility," and the exchange mobility is now the residual.[10] Markandya then goes on to show empirically that it makes a considerable difference (for the Goldthorpe [19] data for Britain) which disaggregation is used.

Markandya's decompositions, though ingenious, pose certain problems: (i) The decompositions are in terms of a mobility (transition) matrix and the steady state distribution vectors induced by this matrix under a Markovian assumption. (Naturally, if such an assumption is not needed, it is better to avoid making it.) (ii) Markandya's procedure requires the analyst to specify a social welfare function to measure *welfare* changes while one might prefer to deal with *income* changes directly. (iii) Total mobility is not additively decomposable into structural and exchange mobility components using his measure. It is rather discomforting that it matters so much empirically which is primary and which is residual.

In our framework, (i) and (ii) are readily handled, while (iii) can be overcome by proceeding as follows. Let us first take the basic concept of *mobility due to a transfer of income*. This involves transfers of income from one individual to another, holding income the same, e.g.,

[9] Markandya [28], pp. 307–308, emphasis in the original.

[10] A *transition matrix* is a nonnegative matrix, the (r, s)th entry of which stands for the probability of moving (or, the proportion of individuals that moved) from social class r to s, $r, s = 1, 2, ..., k$, k being the total number of social classes.

	Change in income distribution	Amount transferred
A:	$(1, 2, 3) \to (1.5, 2, 2.5)$	\$0.50
B:	$(1, 2, 3) \to (3, 2, 1)$	\$2.00
C:	$(1, 2, 3) \to (4, 2, 0)$	\$3.00

(The distributional change in situation B is a special kind of transfer called an *exchange*; it is this notion that has dominated the recent literature on mobility.) Note that in each of these cases, there are winners and losers, and whatever is lost by the losers is won by the winners; what varies in these different situations is the amount transferred from losers to winners.[11] Our basic premise is that the total mobility is strictly increasing in the transferred amount, holding total income constant.

In light of the above treatment, given $x \to y$, $x, y \in \mathbf{R}^n_+$ with $\sum_{j=1}^n y_j \geqslant \sum_{j=1}^n x_j$, we can measure the *total mobility due to transfer of income* as twice the amount lost by losers. That is, letting $\mathscr{L}_n(x, y) \equiv \{j \in \{1, 2, ..., n\} \mid x_j - y_j > 0\}$ be the set of losers in $x \to y$, we define *mobility due to transfer of income in a growing economy* as

$$\mathscr{T}_n(x, y) \equiv 2 \left(\sum_{j \in \mathscr{L}_n(x, y)} (x_j - y_j) \right).$$

("Twice" because every dollar lost by a loser is gained by a winner.) On the other hand, we define *mobility due to transfer of income in a shrinking economy* (that is when $\sum_{j=1}^n y_j < \sum_{j=1}^n x_j$) as

$$\mathscr{T}'_n(x, y) \equiv 2 \left(\sum_{j \in \mathscr{W}_n(x, y)} (y_j - x_j) \right),$$

where $\mathscr{W}_n(x, y) \equiv \{j \in \{1, 2, ..., n\} \mid x_j - y_j < 0\}$ is the set of winners.

Let us now turn to *mobility due to economic growth* (or *contraction*). When growth occurs, winners can win without anyone losing, e.g.,

[11] Here, a "winner" is defined as a person whose income increases and a "loser" is a person whose income decreases. This is thus an *income-sensitive* concept of winners and losers. Alternatively, one might conceive of winners and losers in terms of changes in position. In fact, a referee of this journal justly argued that in the process $(1, 2, 3) \to (5, 4, 3)$ the third individual should be deemed a "loser" since she loses her relative superiority during the transformation. We certainly agree with this argument from the point of view of *ranks*. Yet our entire study is conducted in terms of *income changes*, and therefore it is only consistent to qualify an individual as a "loser" according to that person's own income change. All of our mobility measures deliberately focus only on personal income changes and are thus insensitive to rank reversals in general (recall Axiom 2.4). For a further elaboration on this point, see the last paragraph of Section 5.5.

	Change in income distribution	Amount gained
D:	$(1, 2, 3) \to (1, 2, 3)$	\$0.00
E:	$(1, 2, 3) \to (1, 2, 6)$	\$3.00
F:	$(1, 2, 3) \to (3, 4, 5)$	\$6.00

Although these different cases involve different patterns of gains—in particular, the change in case F is a more egalitarian growth pattern than the change in case E, though both are Pareto improving—what is important for present purposes is the total amount gained or lost. Once again, the basic premise is that total mobility is strictly increasing in the total amount gained, holding the amount lost by losers constant (in this case, at zero). Consequently, given $x \to y$, $x, y \in \mathbf{R}^n_+$, we define the *total mobility due to economic growth* as

$$\mathscr{G}_n(x, y) \equiv \sum_{j=1}^{n} y_j - \sum_{j=1}^{n} x_j.$$

Similarly, *total mobility due to economic contraction* is defined as

$$\mathscr{G}'_n(x, y) \equiv \sum_{j=1}^{n} x_j - \sum_{j=1}^{n} y_j.$$

Let $x, y \in \mathbf{R}^n_+$ be arbitrary and denote $\mathscr{L}_n(x, y)$ and $\{1, 2, ..., n\} \backslash \mathscr{L}_n(x, y)$ by $\mathscr{L}$ and $\mathscr{N}$, respectively. Therefore, in the case of a growing economy, we have

$$\begin{aligned}\sum_{j=1}^{n} |x_j - y_j| &= \sum_{j \in \mathscr{N}} (y_j - x_j) - \sum_{j \in \mathscr{L}} (y_j - x_j)\\ &= \sum_{j \in \mathscr{N}} (y_j - x_j) + \sum_{j \in \mathscr{L}} (y_j - x_j) - 2 \sum_{j \in \mathscr{L}} (y_j - x_j)\\ &= \sum_{j=1}^{n} (y_j - x_j) + 2 \sum_{j \in \mathscr{L}} (x_j - y_j).\end{aligned}$$

Similarly, in a shrinking economy, we have

$$\sum_{j=1}^{n} |x_j - y_j| = \sum_{j=1}^{n} (x_j - y_j) + 2 \sum_{j \in \mathscr{W}_n(x, y)} (y_j - x_j).$$

This, in view of Proposition 3.4, establishes the following observation:

PROPOSITION 4.1. *Let $n \geqslant 1$ and $x, y \in \mathbf{R}^n_+$. If $\sum_{j=1}^{n} y_j \geqslant \sum_{j=1}^{n} x_j$,*

$$d_n(x, y) = \mathscr{T}_n(x, y) + \mathscr{G}_n(x, y),$$

and if $\sum_{j=1}^{n} y_j < \sum_{j=1}^{n} x_j$,

$$d_n(x, y) = \mathscr{T}'_n(x, y) + \mathscr{G}'_n(x, y),$$

where d_n *satisfies Axioms* 2.1–2.4, 2.6 *and* 2.7 (*or, Axioms* 2.1–2.3 *and* 2.4*–2.7).

The per capita and percentage mobility (observed in $x \to y$) due to the transfer of income and that due to economic growth can also be defined as $\mathscr{T}_n(x, y)/n$, $\mathscr{T}_n(x, y)/\sum_{j=1}^{n} x_j$, $\mathscr{G}_n(x, y)/n$ and $\mathscr{G}_n(x, y)/\sum_{j=1}^{n} x_j$, respectively. Proposition 4.1 then also gives the decomposition of our per capita and percentage mobility measures as

$$m_n^\circ(x, y) = \frac{1}{n} \mathscr{T}_n(x, y) + \frac{1}{n} \mathscr{G}_n(x, y) \qquad \text{for all} \quad x, y \in \mathbf{R}_+^n,$$

and

$$p_n^\circ(x, y) = \left(\frac{1}{\sum_{j=1}^{n} x_j}\right) \mathscr{T}_n(x, y) + \left(\frac{1}{\sum_{j=1}^{n} x_j}\right) \mathscr{G}_n(x, y) \qquad \text{for all} \quad x, y \in \mathbf{R}_+^n,$$

respectively. Similar decompositions hold in the case of a shrinking economy.

By virtue of the above decompositions, we conclude that the measures proposed above (d_n°, m_n° and p_n°) satisfy the two basic properties noted in the introduction. First, they are sensitive to transfers to income in the sense that, for given initial and final income totals, the larger (the absolute value of) income changes for the constituent individuals, the more income mobility there is. Second, they are sensitive to improved (or diminished) economic opportunities in the aggregate, i.e. the larger are the gains in income (or losses in income) in the cross-sectional income distributions, the more mobility there is. It is, in this sense, we say that they are *comprehensive* measures of mobility.

5. COMPARISON WITH OTHER MOBILITY CONCEPTS

5.1. *Relation with the Relevant Sociological Literature*

The decomposition analysis given in the preceding section is similar in spirit to the sociological literature where an explicit distinction is made between *structural* and *exchange* mobility (see Bartholomew [6] and the references cited therein). Our approach allows for both kinds of mobility to occur, but with one notable difference: We are concerned not with the movement among unordered *social classes* or *groups* (e.g., manual, non-manual, farm occupations, or each of a number of geographic entities) as the sociologists are, but rather with the *movement among income* (or, earnings, consumption, decile) *levels.*

One way of relating our approach to income mobility to the sociological notion of the movement among social classes is as follows. Given $x \to y$, $x, y, \in \mathbf{R}^n_+$, identify each social class by the name of the individual and an income level. (Hence, each class is either empty or composed of only one agent.) Thus, in the process $x \to y$, the initial and the final set of "social" classes are $K(x)$ and $K(y)$ respectively, where $K(z) \equiv \{(j, z_j) : j = 1, 2, ..., n\}$, $z \in \mathbf{R}^n_+$. Now, we consider a transition matrix $P(x, y) \equiv [p_{rs}]_{n \times n}$ as a non-negative matrix where p_{rs} denotes the proportion of individuals who moved from social class (r, x_r) to (s, x_s), $r, s = 1, 2, ..., n$.

Now, for any given transition matrix P, Bartholomew [6], p. 28, proposes the following mobility measure:

$$l \equiv \sum_{r=1}^{n} \sum_{s=1}^{n} p_r p_{rs} \, |a_r - a_s|, \tag{4}$$

where $p_r > 0$ denotes the initial proportion of people in class r, and a_r stands for the *scale of status* of class r (see Sommers and Conlisk [44, p. 169]). Applying the interpretation outlined in the preceding paragraph, given $x \to y$, (and hence, $P(x, y)$), $x, y \in \mathbf{R}^n_+$, the relevant set of classes are given by $K(x) \cup K(y)$, with income level z_j denoting the scale of status of $(j, z_j) \in K(x) \cup K(y)$. But then since each of these classes is composed of exactly one individual by construction, $p_r = 1/n$, $r = (j, z_j)$, $j = 1, 2, ..., n$, and moreover, in the jth row of $P(x, y)$, we have zeroes everywhere other than the entry in the cell that corresponds to $(j, x_j) \times (j, y_j)$, which is, of course, 1. Therefore, in this case, (4) becomes $l = \frac{1}{n} \sum_{j=1}^{n} |x_j - y_j|$, precisely our per capita mobility measure $m_n^\circ(x, y)$.

Although the above analysis builds a bridge between the theoretical mobility studies in the sociology literature and income mobility, we should note that, to the best of our knowledge, (4) has been neither characterized axiomatically nor proposed as a measure of income mobility, nor would sociologists care to characterize each income level as a separate social class. In this sense, we believe, m_n° retains its originality.

5.2. *Consistency with the Notion of Exchange Mobility*

In much of the empirical literature the original data are transformed into percentile classes and the mobility of individuals among these groupings is examined.[12] The identifying characteristics of this type of analysis are that

[12] See, for instance, the references cited in Atkinson *et al.* [4].

a fixed percentage of the population is assigned to each class, that each class is given a certain rank, and that mobility is measured in terms of the number of ranks moved by each person during the transition period. To be more specific, let us consider k classes, A_j, $j=1, 2, ..., k$, and denote the set of mk dimensional real vectors such that exactly m entries are 1, exactly m entries are 2, and so on up to k, by $X(m, k)$, using the convention of representing an agent whose income belongs to A_j by integer j. (A common example is the case of decile vectors where $k=10$ and A_j corresponds to the jth decile of the income distribution.)

Exchange mobility is typically analyzed using transition matrices rather than distribution vector transformations. Indeed, the information contained in the transformation $x \to y$, $x, y \in X(m, k)$, can be equivalently represented by the transition matrix $P(x, y) = [p_{rs}]_{k \times k}$, where, as in Subsection 5.1, p_{rs} denotes the proportion of the people that were in class r in the distribution x and have now moved to class s. The mobility comparison between, say $x \to y$ and $z \to w$, $z, w \in X(m, k)$, can therefore be studied in terms of the associated transition matrices of these transformations.

How can we compare two transition matrices in the context of exchange mobility? A partial answer to this question is given by Atkinson's notion of *diagonalizing switches* (Atkinson [3]). To define this property, let the transition matrix $P \equiv [p_{rs}]_{k \times k}$ change such that, concerning the rth and sth income classes, the proportions of the individuals remaining in their present positions increase and the proportions of those moving to the other position decrease. That is, let P become $Q \equiv [q_{ij}]_{k \times k}$, where $q_{ij} = p_{ij}$, $i \neq r, r+1, j \neq s, s+1$, and

$$q_{rs} = p_{rs} + \delta, \qquad q_{r, s+1} = p_{r, s+1} - \delta$$

$$q_{r+1, s} = p_{r+1, s} - \delta, \qquad q_{r+1, s+1} = p_{r+1, s+1} + \delta$$

with $\delta > 0$ (Atkinson *et al.* [4, p. 15]).[13] We shall denote this transformation by $P \xrightarrow{ds} Q$. According to [3] and [4], any such diagonalizing switch reduces mobility; that is, if $P \xrightarrow{ds} Q$, then one should conclude unambiguously that Q exhibits less mobility than P.

How does d_n° behave with respect to this particular kind of exchange mobility? To answer this question, we have to distinguish between two kinds of diagonalizing switches. Let $P \xrightarrow{ds} Q$. We shall say that this diagonalizing switch is of *type* 1 if $p_{rs} \notin \text{diag}\, P$, and it is of *type* 2 otherwise. Now, let $x, y, z \in X(m, k)$, $m, k \in \mathbf{Z}_+$, and $P(x, y)$ and $P(x, z)$ be the transition matrices that represent $x \to y$ and $x \to z$, respectively, and further

[13] Four changes are needed to preserve the bistochasticity of $P(x, y)$, (i.e., that $\sum_{r=1}^k p_{rs} = \sum_{s=1}^k p_{rs} = 1$), which arises from the assumption that each class has exactly the same number of members (e.g., 10 percent of the population) at all times.

satisfy $P(x, y) \xrightarrow{ds} P(x, z)$. One can easily show that if the diagonalizing switch is of type 1, $d^{\circ}_{mk}(x, y) = d^{\circ}_{mk}(x, z)$, and if it is of type 2, $d^{\circ}_{mk}(x, y) > d^{\circ}_{mk}(x, z)$.[14] (To see this, notice that $x, y \in X(m, k)$ implies

$$d^{\circ}_{mk}(x, y) = \sum_{i=1}^{mk} \sum_{j=1}^{mk} (mk)\, p_{ij}\, |i-j|,$$

where $P(x, y) = [\,p_{ij}]_{k \times k}$. The proposition can then be verified by direct computation.)

The above observation is hardly surprising. A type 1 diagonalizing switch exchanges the amounts gained (if the switch is above the diagonal) and the amount lost (if it is below the diagonal). Our measure says that if some people move up one class more and an equal number move up one class less, mobility is unchanged. However, with a type 2 diagonalizing switch, there are both winners and losers, and such a switch reduces the number of both while keeping more people in their original classes. Naturally enough, our measure says that mobility is strictly reduced by any such switch.

5.3. *Consistency with the Notion of Monotonicity in Distance*

As defined in Section 4, transfer mobility arises when one person's income gain is another person's income loss, holding total income constant. A key concept regarding mobility in this setting is *monotonicity in distance* which was introduced by Cowell [14]. To define this property, let $x', y' \in \mathbf{R}^n_+$ be defined such that

$$x'_i = x_i + \delta, \qquad y'_i = y_i + \delta$$
$$x'_j = x_j - \delta, \qquad y'_j = y_j - \delta$$

for some $1 \leqslant i, j \leqslant n$, $i \neq j$, and $x'_k = x_k$, $y'_k = y_k$ for all $k \neq i, j$, for an arbitrary small number δ. Let d be a *directed distance function* from $\mathbf{R}^2_+$ to the real line, i.e. an antisymmetric function which is strictly increasing (decreasing) in the first (second) argument such that $d(a, b) = -d(b, a)$ for all $a, b \geqslant 0$. A mobility measure $J: \mathbf{R}^n_+ \times \mathbf{R}^n_+ \to \mathbf{R}_+$ is said to be *monotonic in distance* if

$$\operatorname{sgn}(J(x, y') - J(x, y)) = \operatorname{sgn}(d(y_i, y_j) - d(x_i, x_j)) \tag{5}$$

and

$$\operatorname{sgn}(J(x', y) - J(x, y)) = \operatorname{sgn}(d(x_i, x_j) - d(y_i, y_j)) \tag{6}$$

[14] Of course, this proposition also holds for m_{mk} and p_{mk}.

for some directed distance function d ([14, pp. 138–139]). It is argued that this property "appears to be related to the preference for diagonalizing switches in a transition matrix" (Atkinson *et al.* [4, p. 32]).

The monotonicity in distance axiom as stated in (5) and (6) has one counter-intuitive implication which can be demonstrated by a simple example. Let $x = y = (1, 1)$. Since, for any directed distance function d, $d(x_1, x_2) = d(y_1, y_2)$, (5) implies that $J(x, y') = J(x, y)$. In other words, a mobility measure monotonic in distance has to see the same amount of mobility in $(1, 1) \to (1, 1)$ and in $(1, 1) \to (1+\delta, 1-\delta)$, $\delta \neq 0$, a highly implausible conclusion. This difficulty is, however, clearly an exception, and is remedied immediately if we replace the sgn$(\cdot)$ function above by its restriction to $\mathbf{R}\backslash\{0\}$. It is then easy to verify that our measures d_n°, m_n° and p_n° satisfy this modified version of monotonicity in distance. In this sense, we claim that our measure of mobility is consistent with the basic premise behind the monotonicity in distance property.

5.4. *Relation with Other Axiomatic Approaches*

An earlier attempt to study the measurement of mobility axiomatically is that of Shorrocks [38], who postulated several axioms on the mobility index $M(P)$, P being any transition matrix. (1) (*Normalization*) *Range* $M(\cdot) = [0, 1]$, (2) (*Immobility*) $M(I) = 0$, (3) (*Perfect Mobility*). If all rows of P are identical, $M(P) = 1$, (4) (*Monotonicity*). An increase in an off-diagonal element at the expense of the diagonal strictly increases the value of the index. Shorrocks [38] shows that the normality, perfect mobility and monotonicity axioms are incompatible.

We should note, however, that axioms (1) and (3) appear unexceptionable only in an intergenerational context where the fundamental question is about the notion of *temporal dependence* (that is, the degree to which a son's class is determined by that of his father.) This is not the question that is addressed by the present work. Our concern here is rather with the measurement of how much *movement* has taken place in a given transformation. (See [38] and [6, pp. 24–29] for illuminating discussions of the distinction between these two concepts of mobility.) In this particular context, the "perfect mobility" and "normalization" axioms appear questionable.

To see this, suppose we have a two class society (each class being denoted by 1 and 2) with the transformation $(1, 1, 2, 2) \to (1, 2, 1, 2)$. For such a distributional change, the transition matrix would take the form $\left[\begin{smallmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{smallmatrix}\right]$ which exhibits "perfect mobility" according to Shorrocks' perfect mobility axiom. Yet the process $(1, 1, 2, 2) \to (2, 2, 1, 1)$ shows more movement—this was also noted by Bartholomew [6]—so in this sense the mobility associated with the transition matrix above is not maximal. If we think of these values as incomes, we can go even further. A change like

$(1, 1, 2, 2) \to (3, 3, 0, 0)$ would exhibit more mobility than $(1, 1, 2, 2) \to (1, 2, 1, 2)$ or $(1, 1, 2, 2) \to (2, 2, 1, 1)$, and a change like $(1, 1, 2, 2) \to (6, 0, 0, 0)$ would involve even more mobility than that. Finally, we note that all these changes have held total income constant. If, in addition, we allow for income growth to take place as well, mobility can increase without limit, which is why we also feel that the normalization axiom is unacceptable in the present context.

Another axiomatization of mobility measurement appears in Cowell [14] who characterized a distributional change index $J\colon \mathbf{R}^n_{++} \times \mathbf{R}^n_{++} \to \mathbf{R}$ of the form

$$J(x, y) = H\left(\sum_{i=1}^{n} f(x_i, y_i), \bar{x}, \bar{y}, n\right),$$

where H is strictly increasing in its first argument and $f(a, a) = k_0 + k_1 a$ for some constants k_0 and k_1 ([14, Theorem 1]). It is easy to see that d_n°, m_n° and p_n° are in this characterized class.

Finally, we should mention the recent account given by Shorrocks [42], which explores the compatibility of several mobility indices with twelve candidate mobility axioms. All of these indices are measures of *relative* income mobility in the sense of being scale invariant.[15] Shorrocks also introduces an additional invariance axiom which he calls *intertemporal scale invariance*. (A function $D\colon \mathbf{R}^n_+ \times \mathbf{R}^n_+ \to \mathbf{R}_+$, $n \geqslant 1$, is said to be intertemporally scale invariant if and only if $D(\lambda x, \alpha y) = D(x, y)$ for all $\lambda, \alpha > 0$, $x, y \in \mathbf{R}^n_+$.) In consequence, by intertemporal scale invariance, $x \to y$, $x, y \in \mathbf{R}^n_+$ exhibits *complete immobility* if and only if $x_i / \sum_{j=1}^n x_j = y_i / \sum_{j=1}^n y_j$, $i = 1, 2, ..., n$, while d_n°, m_n° and p_n° record complete immobility if and only if $x_i = y_i$, $i = 1, 2, ..., n$.[16] This observation highlights the basic difference between the approach developed here and the relativist approach of Shorrocks.

5.5. *Relation with Other Approaches to Mobility Measurement*

Much of the mobility literature has used mobility measures based on the assumption that the transition matrix that characterizes the distributional

[15] Examples of relative mobility measures are: the correlation coefficient (McCall [31]), rank correlation (Schiller [35]), Lillard and Willis [26], Gottschalk [20]), average jump in rank (but not income), Hart's index (Hart [22]), Maasoumi and Zandvakili index (Maasoumi and Zandvakili [27]) and Shorrocks' index (Shorrocks [39]).

[16] Thus, an analyst who sees a positive amount of income mobility in $(1, 2) \to (100, 200)$ implicitly rejects intertemporal scale invariance.

change is Markovian.[17] However, the classical Markov assumption of constancy of the transition matrix through time is rejected by empirical studies of Britain, France, and the United States (cf. Shorrocks [37], Atkinson *et al.* [4] and Atoda and Tachibanaki [5]). The approach we have developed above made no such assumption and so is not vulnerable to this criticism.

Another important thrust in the mobility literature has been the welfarist approach pioneered by Atkinson [3] and developed further by Markandya [28, 30], Chakravarty *et al.* [12], Kanbur and Stiglitz [23], Slesnick [43], Atkinson *et al.* [4] and Dardanoni [15]). In this literature, "mobility is seen in terms of its implications rather than from a direct consideration of what is meant by mobility" ([3, p. 71]). We, however, are doing exactly what Atkinson and others are not, namely, considering the meaning and measurement of mobility directly. As stated by Dardanoni [15, p. 374], we aim "to construct summary immobility measures to capture the intuitive descriptive content of the notion [of mobility]."[18]

Finally, we should note that some authors have approached the measurement of mobility by explicitly concentrating on the changes in the relative *ranks* of individuals (cf. King [24] and Chakravarty [9]) which is, in turn, directly related to (and, in fact, sometimes identified by) the concept of horizontal inequity (cf. Plotnick [33]). Our mobility measures do not support this line of reasoning because they are insensitive to rerankings beyond what would be implied by the income changes themselves. To give an example, let $x=(1, 2, 5)$, $y=(1, 4, 5)$ and $z=(3, 2, 5)$. Clearly, $d_3^\circ(x, y)=d_3^\circ(x, z)$ although there is a reranking in one case but not in the other. Indeed, a more rank sensitive measure, for instance the mobility index introduced in King [24], would report a higher mobility in $x \to z$ than in $x \to y$. Nevertheless, the desirability of the sensitivity to reranking remains a subjective issue. Anyone who accepts the weak decomposability axiom (Axiom 2.4*) should retain the position of not assigning any special significance to rerankings. (Indeed, for all $d_3(\cdot, \cdot)$ satisfying Axioms 2.2 and 2.4*, we would have $d_2(x, y)=d_3(x, z)$.)

[17] The Markovian assumption is indeed used rather extensively in the mobility literature. Among those who work in a descriptive model characterized by this assumption are Prais [34], Theil [45], Shorrocks [38], Sommers and Conlisk [44] and Bartholomew [6], and among those using the Markovian assumption in normative models are Markandya [28, 30], Kanbur and Stiglitz [23], Conlisk [13] and Dardanoni [15].

[18] This parallels the different approaches followed in the inequality literature between welfarist and objective approaches. For instance, Sen seems to prefer a more descriptive approach by saying: "...There are some advantages in ... try[ing] to catch the extent of inequality in some objective sense... so that one can distinguish between (a) "seeing" more or less inequality, and (b) "valuing" it more or less in ethical terms" (Sen [36, pp. 2–3]).

6. CONCLUSION

In this paper, we have put forth several desirable axioms for absolute income mobility measures, and characterized an interesting class of such measures. We then refined this class further by means of an additional assumption, and thus obtained a unique total income mobility measure: *the sum of the absolute values of income changes for each individual in the society*. Conversion to a per capita basis allows mobility comparisons for groups consisting of different numbers of people or for surveys consisting of different numbers of respondents. In turn, per capita mobility can be gauged as a proportion of the base income, permitting statements such as "average mobility is 20% of initial income."

We then showed that this mobility measure is additively decomposable into two components, one due to transfers of income from losers to winners, and the other due to growth in the total amount of income. No other exact decomposition of income mobility has appeared in the literature before.

Finally, we compared our measure with others that have appeared in the literature. We observed that although, with some stretch of the model, an approach in the sociological literature comes close to our measure, the fit is by no means exact. In the economics literature, most approaches are Markovian and/or normative, and the descriptive ones are often couched in relative terms as opposed to absolute terms. In this sense, we believe our study complements the existing literature by focusing on the objective measurement of total, per capita and percentage (absolute) income mobility.

7. PROOFS

That

$$\left\{d_n\colon \mathbf{R}_+^{2n} \to \mathbf{R}_+ \,\middle|\, d_n(x, y) = \left(\sum_{j=1}^{n} |x_j - y_j|^\alpha\right)^{1/\alpha} \forall x, y \in \mathbf{R}_+^n, n \geqslant 1\right\}$$

satisfies Axioms 2.1–2.6 and Axiom 2.4* for any $\alpha > 0$ can easily be verified. We shall now show that this is the only class that satisfies these axioms. (In what follows, we continue our convention of referring to $\{d_n\colon \mathbf{R}_+^{2n} \to \mathbf{R}_+ \,|\, n \geqslant 1\}$ as simply d_n.)

7.1. *Proof of Theorem* 3.1

We shall first establish that the functions F_n and F_{n+1}, $n \geqslant 2$, satisfy the following special case of the *generalized associativity equation* (Aczel [1, p. 310, Eq. (1)]):

$$F_{n+1}(F_n(a, b), c) = F_{n+1}(a, F_n(b, c)) \qquad \forall a, b, c \geqslant 0. \tag{7}$$

Fix $n \geqslant 2$ and pick arbitrary $a, b, c \geqslant 0$. Since Axiom 2.1 implies that d_k is surjective on $\mathbf{R}_+^{2k}$ for any $k \geqslant 1$, we can choose $x_1, x_{n+1}, y_1, y_{n+1} \geqslant 0$ such that $d_1(x_1, y_1) = a$ and $d_1(x_{n+1}, y_{n+1}) = c$, and $x^2, y^2 \in \mathbf{R}_+^{n-1}$ such that $d_{n-1}(x^2, y^2) = b$. Then, by Axiom 2.4,

$$\begin{aligned} &d_{n+1}((x_1, x^2, x_{n+1}), (y_1, y^2, y_{n+1})) \\ &\quad = F_{n+1}(d_n((x_1, x^2), (y_1, y^2)), d_1(x_{n+1}, y_{n+1})) \\ &\quad = F_{n+1}(F_n(d_1(x_1, y_1), d_{n-1}(x^2, y^2)), c) \\ &\quad = F_{n+1}(F_n(a, b), c) \end{aligned}$$

and

$$\begin{aligned} &d_{n+1}((x_1, x^2, x_{n+1}), (y_1, y^2, y_{n+1})) \\ &\quad = F_{n+1}(d_1(x_1, y_1), d_n((x^2, x_{n+1}), (y^2, y_{n+1}))) \\ &\quad = F_{n+1}(a, F_n(d_{n-1}(x^2, y^2), d_1(x_{n+1}, y_{n+1}))) \\ &\quad = F_{n+1}(a, F_n(b, c)). \end{aligned}$$

We may therefore conclude that (7) holds for all $n \geqslant 2$.

We proceed with a number of lemmata.

LEMMA 7.1. *Let d_n satisfy Axioms 2.1, 2.2 and 2.4. Then,*

$$d_n(\mathbf{1}_n, \mathbf{1}_n) = F_n(0, 0) = 0 \qquad \forall n \geqslant 2.$$

Proof. For any $n \geqslant 2$, by Axioms 2.1 and 2.2, $d_n(\mathbf{1}_n, \mathbf{1}_n) = d_n(\mathbf{0}_n, \mathbf{0}_n) = \lambda d_n(\mathbf{0}_n, \mathbf{0}_n)$ for all $\lambda > 0$, where $\mathbf{0}_n = (0, ..., 0) \in \mathbf{R}^n$. Thus, $d_n(\mathbf{1}_n, \mathbf{1}_n) = d_n(\mathbf{0}_n, \mathbf{0}_n) = 0$. (We have, of course, $d_1(1, 1) = 0$ analogously.) Moreover, by Axiom 2.4,

$$F_n(0, 0) = F_n(d_{n_1}(\mathbf{1}_{n_1}, \mathbf{1}_{n_1}), d_{n_2}(\mathbf{1}_{n_2}, \mathbf{1}_{n_2})) = d_n(\mathbf{1}_n, \mathbf{1}_n) = 0,$$

with $n_1 + n_2 = n$, and hence the lemma. ∎

LEMMA 7.2. *Let d_n satisfy Axioms* 2.1, 2.2, 2.4, *and* 2.6. *Then, for all* $n \geqslant 2$,

$$F_n(a, 0) = a \qquad \forall a \geqslant 0.$$

Proof. Fix $n \geqslant 2$ and pick an arbitrary $a > 0$. By surjectivity of d_1, we can choose $x_1, y_1, z_1, w_1 \geqslant 0$ such that $d_1(x_1, y_1) = a$ and $d_1(z_1, w_1) = F_n(a, 0)$. By Axiom 2.4 and Lemma 7.1, we have

$$\begin{aligned} d_{n+1}((x_1, \mathbf{1}_n), (y_1, \mathbf{1}_n)) &= F_{n+1}(d_1(x_1, y_1), d_n(\mathbf{1}_n, \mathbf{1}_n)) \\ &= F_{n+1}(a, 0) = F_{n+1}(a, F_n(0, 0)). \end{aligned}$$

But, by (7), Lemma 7.1 and Axiom 2.4,

$$\begin{aligned} F_{n+1}(a, F_n(0, 0)) &= F_{n+1}(F_n(a, 0), 0) \\ &= F_{n+1}(d_1(z_1, w_1), 0) \\ &= F_{n+1}(d_1(z_1, w_1), d_n(\mathbf{1}_n, \mathbf{1}_n)) \\ &= d_{n+1}((z_1, \mathbf{1}_n), (w_1, \mathbf{1}_n)) \end{aligned}$$

so that we must have

$$d_{n+1}((x_1, \mathbf{1}_n), (y_1, \mathbf{1}_n)) = d_{n+1}((z_1, \mathbf{1}_n), (w_1, \mathbf{1}_n)).$$

Axiom 2.6, therefore, yields that

$$a = d_1(x_1, y_1) = d_1(z_1, w_1) = F_n(a, 0),$$

and the claim follows. ∎

LEMMA 7.3. *Let d_n satisfy Axioms* 2.1, 2.2, 2.4, *and* 2.6. *Then, for all* $n \geqslant 2$, $F_n = F_{n+1}$.

Proof. Take any $n \geqslant 2$, and notice that by Lemma 7.2, and (7), we have

$$F_n(a, b) = F_{n+1}(F_n(a, b), 0) = F_{n+1}(a, F_n(b, 0)) = F_{n+1}(a, b)$$

for any $a, b \geqslant 0$. ∎

Given Lemma 7.3, (7) therefore, reads as

$$F(F(a, b), c) = F(a, F(b, c)) \qquad \forall a, b, c \geqslant 0,$$

where $F = F_n$, $n \geqslant 2$. This is, of course, nothing but the classical *associativity equation* ([1, pp. 253–272]). On the other hand, for any $a, a', b, b' \geqslant 0$, Axioms 2.4 and 2.6 imply that

$$F(a, b) = F(a, b') \Rightarrow b = b' \qquad \text{and} \qquad F(a, b) = F(a', b) \Rightarrow a = a',$$

that is F is *reducible from both sides* ([1, p. 255]). Thus, we can apply the theorem in [1, p. 256] to get

$$F(a, b) = f(f^{-1}(a) + f^{-1}(b)) \qquad \forall a, b \geqslant 0, \tag{8}$$

for some strictly monotonic and continuous function $f: \mathbf{R}_+ \to \mathbf{R}_+$. (By Lemma 7.1 and (8), $f^{-1}(0) = f^{-1}(F(0, 0)) = 2f^{-1}(0)$ so that we have $f^{-1}(0) = f(0) = 0$. Of course, this verifies that f is strictly increasing.) But one can easily show that Axiom 2.1 implies the linear homogeneity of F, and consequently, by an immediate application of Theorem 2.2.1 of Eichhorn [17, p. 32], we have either $F(a, b) = Aa^r b^{1-r}$ for all $a, b \geqslant 0$, for some $A > 0$ and $0 < r < 1$, or $F(a, b) = (Aa^\alpha + Bb^\alpha)^{1/\alpha}$ for all $a, b \geqslant 0$, for some $A, B > 0$ and $\alpha \neq 0$. The former is impossible in view of Lemma 7.2, and thus the latter must be the case. By symmetry, Lemma 7.2 and continuity of F at the origin however, we must further have $A = B = 1$ and $\alpha > 0$ in the latter case. Therefore, for all $n \geqslant 2$,

$$F(a, b) = F_n(a, b) = (a^\alpha + b^\alpha)^{1/\alpha} \qquad \forall a, b \geqslant 0$$

must hold for some $\alpha > 0$. But then successively applying Axiom 2.4, we obtain

$$d_n(x, y) = \left(\sum_{j=1}^{n} d_1(x_j, y_j)^\alpha \right)^{1/\alpha} \qquad \text{for all} \quad x, y \in \mathbf{R}_+^n, \quad n \geqslant 2,$$

for some $\alpha > 0$. The proof of Theorem 3.1 will, therefore, be complete if we can show that $d_1(a, b) = |a - b|$ for all $a, b \geqslant 0$. But, Axiom 2.2, Axiom 2.1 and Axiom 2.3, it follows that

$$d_1(a, b) = \begin{cases} d_1(a - b, 0), & \text{if} \quad a \geqslant b \\ d_1(0, b - a), & \text{if} \quad a < b \end{cases}$$

$$= \begin{cases} |a - b| \; d_1(1, 0, & \text{if} \quad a \geqslant b \\ |a - b| \; d_1(0, 1), & \text{if} \quad a < b \end{cases} \quad = \quad |a - b|,$$

for any $a, b \geqslant 0$.

7.2. *Proof of Theorem* 3.2.

We start with the following useful lemma.

LEMMA 7.4. *Let d_n satisfy Axioms* 2.1, 2.4* *and* 2.5. *For any* $x, y \in \mathbf{R}^n_+$,

$$d_n(x, y) = G_2(G_{n-1}(d_1(x_1, y_1), ..., d_1(x_{n-1}, y_{n-1})), d_1(x_n, y_n)).$$

Proof. Proof is by induction on n. Let $n = 3$ and let $x, y \in \mathbf{R}^3_+$ be arbitrary. By surjectivity of d_1, there exist $z, w \in \mathbf{R}_+$ such that $d_2((x_1, x_2), (y_1, y_2)) = d_1(z, w)$, and by Axiom 2.4*,

$$d_1(z, w) = G_2(d_1(x_1, y_1), d_1(x_2, y_2)). \tag{9}$$

By Axiom 2.5, Axiom 2.4* and (9),

$$\begin{aligned} d_3(x, y) &= d_2((z, x_3), (w, y_3)) \\ &= G_2(d_1(z, w), d_1(x_3, y_3)) \\ &= G_2(G_2(d_1(x_1, y_1), d_1(x_2, y_2)), d_1(x_3, y_3)). \end{aligned}$$

Now let $n = k$ and assume that, for any $x, y \in \mathbf{R}^k_+$,

$$d_k(x, y) = G_2(G_{k-1}(d_1(x_1, y_1), ..., d_1(x_{k-1}, y_{k-1})), d_1(x_k, y_k)).$$

To complete the proof, let $x, y \in \mathbf{R}^{k+1}_+$ and notice that, again by the surjectivity of d_{k-1}, we can choose $z, w \in \mathbf{R}^{k-1}_+$ such that

$$d_k((x_1, ..., x_k), (y_1, ..., y_k)) = d_{k-1}(z, w). \tag{10}$$

But by Axiom 2.5, induction hypothesis, Axiom 2.4*, (10), and Axiom 2.4* again,

$$\begin{aligned} d_{k+1}(x, y) &= d_k((z, x_{k+1}), (w, y_{k+1})) \\ &= G_2(G_{k-1}(d_1(z_1, w_1), ..., d_1(z_{k-1}, w_{k-1})), d_1(x_{k+1}, y_{k+1})) \\ &= G_2(d_{k-1}(z, w), d_1(x_{k+1}, y_{k+1})) \\ &= G_2(d_k((x_1, ..., x_k), (y_1, ..., y_k)), d_1(x_{k+1}, y_{k+1})) \\ &= G_2(G_k(d_1(x_1, y_1), ..., d_1(x_k, y_k)), d_1(x_{k+1}, y_{k+1})), \end{aligned}$$

and the lemma is proved. ▮

We shall next determine the functional form of G_2. To this end, we start with the following observation:

LEMMA 7.5. *Let d_2 satisfy Axioms* 2.1, 2.2 *and* 2.4*. *Then,* $G_2(0, 0) = 0$.

Proof. Similar to Lemma 7.2. ▮

Now, by the surjectivity of d_3 and Lemma 7.4,

$$G_3(a, b, c) = G_2(G_2(a, b), c) \qquad \forall a, b, c \geqslant 0. \tag{11}$$

But by symmetry of G_3 and Lemma 7.4,

$$G_3(a, b, c) = G_3(b, c, a) = G_2(G_2(b, c), a) = G_2(a, G_2(b, c)) \qquad \forall a, b, c \geqslant 0,$$

and combining this with (11),

$$G_2(G_2(a, b), c) = G_2(a, G_2(b, c)) \qquad \forall a, b, c \geqslant 0. \tag{12}$$

But then, proceeding as in the proof of Theorem 3.1, we must have

$$G_2(a, b) = f(f^{-1}(a) + f^{-1}(b)) \qquad \forall a, b \geqslant 0, \tag{13}$$

for some strictly monotonic and continuous function $f: \mathbf{R}_+ \to \mathbf{R}_+$. By Lemma 7.5 and (13), $f^{-1}(0) = f^{-1}(G_2(0, 0)) = 2f^{-1}(0)$ so that we have $f^{-1}(0) = f(0) = 0$, and f is strictly increasing. But, by Axiom 2.1 and Axiom 2.4*, G_2 is linearly homogeneous, and therefore, we obtain

$$G_2(a, b) = (a^\alpha + b^\alpha)^{1/\alpha} \qquad \forall a, b \geqslant 0$$

for some $\alpha > 0$. Therefore, by successively applying Lemma 7.4, we have

$$\begin{aligned} d_n(x, y) &= \underbrace{G_2(G_2(\ldots G_2}_{n-1 \text{ times}}(d_1(x_1, y_1), d_1(x_2, y_2)), \ldots, \\ &\qquad d_1(x_{n-1}, y_{n-1})), d_1(x_n, y_n)) \\ &= \left(\sum_{j=1}^{n} d_1(x_j, y_j)^\alpha \right)^{1/\alpha}, \end{aligned}$$

for all $x, y \in \mathbf{R}^n_+$, $n \geqslant 2$. Since Axioms 2.1–2.3 imply that $d_1(a, b) = |a - b|$ for all $a, b \geqslant 0$, the proof of the theorem is complete.

7.3. *Proof of Theorem* 3.3

Define $G(a, b) = f(f^{-1}(a) + f^{-1}(b))$ for all $a, b \geqslant 0$. Since $f(0) = 0$, we must have

$$G(a, 0) = a \qquad \text{and} \qquad G(0, b) = b \qquad \forall a, b \geqslant 0. \tag{14}$$

Pick any $a, b, b' \geqslant 0$ and choose $x, y, x', y' \in \mathbf{R}^2_+$ such that $x_1 = x'_1$, $y_1 = y'_1$, $d_1(x_1, y_1) = b$, $d_1(x_2, y_2) = b'$ and $d_1(x'_2, y'_2) = a$. By Axiom 2.7,

$$d_2(x, y) - d_2(x, (x_1, y_2)) = d_2(x', y') - d_2(x', (x'_1, y'_2)),$$

and this yields

$$G(b, b') - G(0, b') = G(b, a) - G(0, a)$$

since $d_1(t, t) = 0$ for all $t \geqslant 0$, by hypothesis. By (14), this gives us $G(b, b') - G(b, a) = b' - a$. By the symmetry of G, therefore, we may conclude that

$$G(b, b') - G(b, a) = b' - a \qquad \forall a, b, b' \geqslant 0.$$

For any $a \geqslant 0$ and $h > -a$, choosing $b' = a + h$, we obtain

$$G(a+h, b) - G(a, b) = h \qquad \forall a, b \geqslant 0, \quad h \geqslant -a.$$

But then

$$\frac{G(a+h, b) - G(a, b)}{h} = 1 \qquad \forall a, b > 0, \quad h \geqslant -a$$

so that, for any $a, b > 0$,

$$\lim_{h \to 0} \left(\frac{G(a+h, b) - G(a, b)}{h} \right) = 1$$

establishing the existence of the first partial derivatives of G with $\partial G(a, b)/\partial a = 1$. Integrating with respect to a, we have $G(a, b) = a + \phi(b)$, $a > 0$, $\phi(b)$ being the integration constant for each $b > 0$. By the symmetry of G, we must also have $G(a, b) = b + \phi(a)$, $a, b > 0$, so that $\phi'(t) = 1$ for all $t > 0$. Then, $\phi(t) = t + \delta$, $t > 0$, where δ is the integration constant. Consequently, in view of (14),

$$G(a, b) = \begin{cases} a + b + \delta & \text{if} \quad a > 0, b > 0 \\ a + b, & \text{otherwise.} \end{cases}$$

But recalling that f must be continuous at the origin, $\delta = 0$ obtains. In conclusion, $G(a, b) = a + b$ for all $a, b \geqslant 0$. But then,

$$f^{-1}(a+b) = f^{-1}(a) + f^{-1}(b) \qquad \forall a, b > 0,$$

that is, f^{-1} satisfies *Cauchy*'s basic functional equation. Therefore, $f^{-1}(a) = \tau a$ for all $a > 0$ must be true for some $\tau > 0$, and Proposition 3.3 follows.

REFERENCES

1. J. Aczel, "Lectures on Functional Equations," Academic Press, London, 1966.
2. A. B. Atkinson, On the measurement of inequality, *J. Econ. Theory* **2** (1970), 244–263.
3. A. B. Atkinson, The measurement of economic mobility, *in* "Inkomens verdeling en Openbare Financien" (P. J. Eggelshoven and L. J. van Gemerden, Eds.), Het Spectrum, Leiden, 1981.
4. A. B. Atkinson, F. Bourguignon, and C. Morisson, "Empirical Studies of Earnings Mobility," Harwood Academic Publishers, 1992.
5. N. Atoda and T. Tachibanaki, Earnings distribution and inequality over time: education versus relative position and cohort, *Int. Econ. Rev.* **32** (1991), 475–489.
6. D. J. Bartholomew, "Stochastic Models for Social Processes," Wiley, London, 1982.
7. R. Boudon, "Mathematical Structures of Social Mobility," Elsevier, Amsterdam, 1973.
8. F. Bourguignon, Decomposable income inequality measures, *Econometrica* **47** (1979), 901–920.
9. S. J. Chakravarty, Normative indices for measuring social mobility, *Econ. Lett.* **15** (1984), 175–180.
10. S. J. Chakravarty, "Ethical Social Index Numbers," Springer-Verlag, Berlin, 1990.
11. S. J. Chakravarty and B. Dutta, A note on measures of distances between income distributions, *J. Econ. Theory* **41** (1987), 185–189.
12. S. J. Chakravarty, B. Dutta, and J. A. Weymark, Ethical indices of income mobility, *Soc. Choice Welfare* **2** (1985), 1–21.
13. J. Conlisk, Ranking mobility matrices, *Econ. Lett.* **29** (1989), 231–235.
14. F. Cowell, Measures of distributional change: An axiomatic approach, *Rev. Econ. Stud.* **52** (1985), 135–151.
15. V. Dardanoni, Measuring social mobility, *J. Econ. Theory* **61** (1993), 372–394.
16. U. Ebert, Measures of distance between income distributions, *J. Econ. Theory* **32** (1984), 266–274.
17. W. Eichhorn, "Functional Equations in Economics," Addison–Wesley, Reading, MA, 1978.
18. G. S. Fields and E. A. Ok, "The Meaning and Measurement of Economic Mobility," CAE Working Paper No. 94-01, Cornell University, 1994.
19. J. M. Goldthorpe, "Social Mobility and Class Structure in Britain," Oxford Univ. Press, Oxford, 1980.
20. P. Gottschalk, Earnings mobility: Permanent change or transitory fluctuations?, *Rev. Econ. Stat.* (1982), 450–456.
21. N. Haaser and J. Sullivan, "Real Analysis," Dover, New York, 1971.
22. P. E. Hart, The statics and dynamics of income distributions: A Survey, *in* "The Statics and Dynamics of Income" (N. A. Klevmarken and J. A. Lybeck, Eds.), Tieto, Clevedon, 1981.
23. S. M. Kanbur and J. Stiglitz, Intergenerational mobility and dynastic inequality, mimeo, Princeton University, 1986.
24. M. A. King, An index of inequality: With applications to horizontal equity and social mobility, *Econometrica* **51** (1983), 99–115.
25. S.-Ch. Kolm, Unequal inequalities I, *J. Econ. Theory* **12** (1976), 416–442.
26. L. A. Lillard and R. J. Willis, Dynamic aspects of earnings mobility, *Econometrica* **46** (1978), 985–1012.
27. E. Maasoumi and S. Zandvakili, A class of generalized measures of mobility with applications, *Econ. Lett.* **22** (1986), 97–102.
28. A. Markandya, Intergenerational exchange mobility and economic welfare, *Europ. Econ. Rev.* **17** (1982), 307–324.

29. A. Markandya, The measurement of earnings mobility among occupational groups, *Scot. J. Polit. Econ.* **29** (1982a), 75–88.
30. A. Markandya, The welfare measurement of changes in economic mobility, *Economica* **51** (1984), 457–471.
31. J. J. McCall, "Income Mobility, Racial Discrimination and Economic Growth," D. C. Heath, Lexington, 1973.
32. T. Mitra and E. A. Ok, "The Measurement of Income Mobility: A Partial Ordering Approach," *Econ. Theory*, forthcoming, (1997).
33. R. D. Plotnick, The concept and measurement of horizontal inequity, *J. Pub. Econ.* **17** (1982), 373–391.
34. S. J. Prais, Measuring social mobility, *J. Roy. Statist. Soc. A* **118** (1955), 56–66.
35. B. Schiller, Relative earnings mobility in the U.S., *Amer. Econ. Rev.* (1977), 926–941.
36. A. K. Sen, "On Economic Inequality," Oxford Univ. Press, Oxford, 1973.
37. A. F. Shorrocks, Income mobility and the Markov assumption, *Econ. J.* **86** (1976), 556–578.
38. A. F. Shorrocks, The measurement of mobility, *Econometrica* **46** (1978), 1013–1024.
39. A. F. Shorrocks, Income inequality and income mobility, *J. Econ. Theory* **19** (1978a), 376–393.
40. A. F. Shorrocks, On the distance between income distributions, *Econometrica* **50** (1982), 1337–1339.
41. A. F. Shorrocks, Inequality decomposition by population subgroups, *Econometrica* **52** (1984), 1369–1385.
42. A. F. Shorrocks, On the Hart measure of income mobility, *in* "Industrial Concentration and Economic Inequality" (M. Casson and J. Creedy, Eds.), Edward Elgar, 1993.
43. D. T. Slesnick, Welfare distributional change and the measurement of social mobility, *Rev. Econ. Statist.* (1986), 586–593.
44. P. M. Sommers and J. Conlisk, Eigenvalue immobility measures for Markov chains, *J. Math. Soc.* **6** (1978), 253–276.
45. H. Theil, "Statistical Decomposition Analysis," North-Holland, Amsterdam, 1972.